Introduction to Mathematical Modeling of Crop Growth

How the Equations are Derived and Assembled into a Computer Model

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BrownWalker Press
Boca Raton • 2006

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USA • 2006

ISBN: 1-58112-999-8 (paperback)
ISBN: 1-58-112-998-X (ebook)

BrownWalker.com

to Jennifer Low J.L.

with all my love

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Preface

Why learning mathematical modeling can be difficult

I believe there are three reasons why most agriculturists find mathematical modeling difficult to learn. Firstly, agriculturists are trained with little emphasis in applied mathematics. Consequently, agriculturists do not know how mathematics can be used or applied in their field. Some even believe that mathematics is inconsequential in agriculture. Agriculture, they say, is essentially biology and chemistry; mathematics is only useful for basic tasks such as to calculate fertilizer or formulation rates.

Secondly, agriculturists are trained very little, if at all, in computer programming. Modeling is not about computers or information technology, as we will later discuss in section 1.2 (item 5). Computers, however, are invaluable tools which allow us to quickly develop, test and modify our models. But to do all this requires some programming skills.

Thirdly, mathematical modeling requires abstract or conceptual thinking – something we do not do well instinctively. Contrary to common belief, mathematical modeling is not about solving equations. It is about describing and translating a real system into a mathematical form. It is about finding the patterns in the behavior or action of the system, and translating those patterns into an equation or set of equations. To do this, however, depends strongly on our ability to generalize and conceptualize the system into a simpler form. Keith Devlin, in his book "*The math gene: how mathematical thinking evolved and why numbers are like gossip*" (Basic Books, 2000), says that mathematics is more than just numbers and equations; mathematics is the science of patterns which involves thinking abstractly about things, ideas and situations.

Consequently, most people (including agriculturists) find mathematical modeling difficult and daunting. They mistakenly believe that it is a field limited only to those who are naturally gifted in mathematics. This situation is not helped when almost all agriculture modeling books read more like a "recipe book of equations". These books are permeated with equations, numbers and strange-looking symbols, but they offer little or no explanation as to how these equations were derived. It is no surprise then that most agriculturists often wonder how these equations were derived – as if by "magic', or the product of some math genius.

Why this book is written

I wrote this book because I wanted a book that explains clearly and in depth how mathematics can be applied in agriculture. I wanted a book that takes the trouble to explain how each equation is derived and how it is used. And finally, I wanted a book that shows how these equations work together and are finally assembled into a computer program for model simulations. I believe my wish list is not unique; it is shared by other agriculturists who desire a book that speaks *to* them rather *at* them. Mathematical modeling need not be difficult, but we require a book that does not only list the equations one-by-one but shows how they are derived and used.

What this book is about

This book is to show how mathematics is applied in agriculture, in particular to modeling the growth and yield of a generic crop. Principles learn from the growth and yield of a generic crop can then be applied to "real" crops such as maize, rice and oil palm. This book explains how each equation is derived and used. Finally, this book shows how all the equations work together for model simulations by assembling them into a C++ computer program.

What this book is <u>not</u> about

This book is not meant to be a comprehensive discussion about all modeling approaches or widely-used crop models. Rather than covering many modeling approaches but in superficial detail, this book selects one or two widely-used modeling approaches and discusses about them in detail. By discussing a particular method in detail, this equips readers with the necessary principles required when they encounter other modeling approaches or crop models.

This book is also not about C++ computer programming. This would require a separate book. Nonetheless, the computer programs in this book are written deliberately in a simple manner so that readers only require a basic knowledge in C++ to understand them.

Finally, this book is not a *leisure* reading material. Although, I have tried my best to explain how each equation is derived, readers are still expected to roll up their sleeves and do some hard work. Readers must think deeply and thoroughly, ask questions, and refer to other reading materials for further clarification or for a deeper understanding.

Who should read this book

This book is intended for senior undergraduates, postgraduates, academicians and scientists in the field of agriculture. Basic knowledge in mathematics, in particular algebra, trigonometry and calculus, is required. Additional knowledge in basic C++ programming will be helpful to build the computer model.

Synopsis

This book should be read in sequential order: Chapter 1 to 8. Topics discussed in one chapter often refer to topics from previous chapters. Readers who are already familiar with the principles of mathematical modeling, however, may skip Chapter 1. This chapter discusses about the meaning, usefulness and many types of mathematical models. It also discusses about modeling methodology.

Chapter 2 and onwards are specific to agriculture. Chapter 2 discusses about the simulation of several weather components such as solar irradiance, vapor pressure, wind speed and air temperature. These components are very important as they drive several plant and soil processes such as evapotranspiration, photosynthesis, soil water balance, respiration and growth. Chapter 3 discusses about the interception of solar radiation by plant canopies. Solar radiation is the major energy source for plants, in particular for photosynthesis (Chapter 6), and the assimilates produced by photosynthesis are used for plant maintenance and growth which are both covered in Chapter 7. Chapter 4 and 5 discuss about the water balance. Chapter 4 is the most detailed chapter which discusses about the various heat fluxes from the soil and plant. The final chapter (Chapter 8) shows how the various equations from previous chapters are assembled into a C++ computer program. This computer program is named *Gg* which stands for **G**eneric crop **g**rowth model. This chapter also shows the results of some model simulations.

Acknowledgment and feedback

I am grateful to Dr. Ian E. Henson (Malaysian Palm Oil Board) and Goh Kah Joo (Applied Agriculture Research Sdn. Bhd.) for reviewing this book; any mistakes, however, are mine alone. I am also grateful to Jennifer Low for her love and support.

All source codes and the sample weather data in this book can be downloaded for free from: www.agri.upm.edu.my/~chris. I appreciate any feedback to this book, and I can be contacted by email at: chris@agri.upm.edu.my.

Christopher Teh Boon Sung
Serdang; 26 Feb. 06

Chapter 1. Mathematical modeling

1.1 What is a mathematical model?

A *model* is a simplified representation of a real system. There are two important concepts in this definition. Firstly, *system* is a group of objects (components or factors) that interact with one another in an organised manner, and the net result of their interactions produces the system's behavior, function and purpose. Examples of systems in agriculture are photosynthesis, soil water flow, crop growth and yield, and the interception of sunlight by the plant canopies.

Secondly, when a system is represented or described in a simpler form known as a model, the model becomes a tool for us to understand the system by helping us to sift through its complexity and to focus on the important, relevant aspects. Models can be of many types:

1. pictorial (*e.g.*, illustrations, diagrams and flowcharts);
2. conceptual or verbal (descriptions in a natural language);
3. physical (replica or mock-up of the system such as a scale airplane model for wind tunnel experiments, and the helical, double-stranded structure of the DNA molecule); and
4. mathematical (Haefner, 1996).

Consequently, a *mathematical model* is one type of simplified representation of a real system. It describes the system using mathematical principles in the form of an equation or a set of equations. For example, in a controlled field experiment by Pandey *et al.* (2000), the maize yield (Y; kg ha^{-1}) responds to the nitrogen fertilizer rate (N; kg ha^{-1}) by the following mathematical/statistical relationship:

$$Y = 1143 + 31.7N - 0.084N^2 \qquad [1.1]$$

which reveals that with increasing nitrogen rates, maize yield increases but with diminishing returns, and that past a certain threshold of nitrogen level (189 kg ha^{-1}), the maize yield will instead start to decrease (probably due to nitrogen toxicity).

1.2 Uses of mathematical models

Mathematical models are developed primarily because we want to solve one or more problems in a real system. For example: how do we increase the growth and yield of the rice crops? Why are the current conservation practices not effective in controlling the soil and water losses from the soil? And what is the effect on oil palm yield due to higher planting density on peat soils?

As compared to other model types (*e.g.*, pictorial, conceptual and physical), mathematical models are often the most useful type. This is because they not only tell us what are the essential components of a real system, but more importantly, they also tell us how and to what degree do each of these components interact with each other. Besides helping us to comprehend the system (*i.e.*, the "what, why and how" of a system), mathematical models exclusively allow us, in an organised manner, to predict (*i.e.*, forecast or estimate the outcome) and control the system (*i.e.*, constrain or manipulate the system's behavior to the desirable outcome). Consequently, mathematical models provide us a tool to solve problems. Specifically, the following are some ways, as adapted from France and Thornley (1984), in which mathematical models can contribute to agricultural research and activities:

1. Modeling helps us to understand, predict and control a system in a more organised or methodological manner because models provide: a) a quantitative description of the system, and b) a way of bringing together knowledge about the parts to give a coherent and holistic view of the system.

2. Models can help to identify areas where knowledge is lacking, and can help to stimulate new ideas or approaches for research.

3. Modeling leads to less experimentation by trial-and-error, and can reduce the time and cost of experiments. This is particularly relevant for long term growing crops such as oil palm which can take many years to mature. Consequently, actual field experiments for such crops are expensive, time-consuming, and must be planned with great care so that useful results are ensued from such long term experiments. Another good example is the work by Peng *et al.* (2004), who produced a China "super" rice hybrid that gave yields up to 12 ton ha^{-1}, breaking the yield ceiling of 10 ton ha^{-1}. Peng *et al.* used a mathematical model to identify the characteristics of a rice plant that were theoretically efficient in photosynthesis, growth and grain production. They then started a rice breeding project to produce rice hybrid varieties that had these idealized plant characteristics.

4. Models can be used to identify interesting and stimulating areas of research, and those short listed can later be implemented in actual experiments.

5. The predictive power of models can be used to answer various "What if?" scenarios such as the yield response of maize under different water management practices, or the effect on oil palm growth and yield on compact, steep lands. Without the use of

models, we must conduct actual experiments to predict the outcome of such “What if?” scenarios which can be expensive and time-consuming. In this manner, models can help in the research and development as well as in the management and planning of agriculture activities.

6. Models can complement and add value to actual experiments. Interpretation of results from experiments can sometimes be aided using a model. For example, a soil water and crop growth model can be used to explain the observed yield response of maize in an experiment with different water management practices.
7. In special circumstances, models can replace experiments to study the effects of certain factors or conditions that otherwise cannot be studied in actual experiments due to high costs, great length of time, high risks or technical difficulties. Nonetheless, it is very important to stress that these are special circumstances when models do replace the necessity of doing actual experiments. In normal conditions, mathematical models are not substitutes for doing actual experiments. In fact, model development relies on results from actual experiments for a deeper understanding of the system so that, in turn, more accurate and comprehensive models can be developed.
8. Modeling encourages collaboration among researchers from various fields of expertise because a system consists of various components, where the study of these different components often requires separate fields of study.

1.3 Characteristics of mathematical models

1) Models are incomplete description of whole systems

Because a model is a simpler representation of a real system, it is actually an *incomplete* or imperfect description of the system. There is always some loss of information when a real system is translated into a model. A model does not tell us the whole picture of the system, only the essential parts that are important to explain the system’s behavior, function and purpose. This incomplete depiction of a system means that the estimations or simulations from a mathematical model are subject to error.

The degree of modeling error depends on how much information is lost; that is, how much of the essential parts of the system that we have ignored in our models. It also depends on how much we have under- or overestimated the effects of the system components. The key here is then for us to be able to capture and summarize the essential information of the real system into a model. We have to identify the essential components of the real system, and to describe adequately how and to what degree each of these components interacts with one another into our model. Consequently, what we achieve at the end is a model that is simpler than the real system but immensely useful because it helps us to study the system, and additionally, to control and anticipate the system's behavior with the desired detail and accuracy.

We might then be tempted to develop a model that is complete, that perfectly represents a real system in every way. But such a model, perfect though it may be, is of no use to us because what we have actually done is not the development of a tool to aid our understanding but the perfect duplication or "cloning" of the actual system itself. So, in the end, what we have is a model that is just as complicated and intricate as the actual system itself. We would be no better using such a model to understand the system. In the other extreme, we must be careful too that we do not develop models that are too simple or incomplete, that vital aspects of the system are ignored or described inadequately, and that our understanding of the system become deficient, or worse, wrong.

2) Models are built from assumptions

Real systems are often complex and we need assumptions to simplify them for our understanding. Without the use of assumptions, we might fail to develop a model because there is still too much complexity or uncertainty about the system. In other words, assumptions help us to grasp the system so that we are able to summarize the system into a model.

However, there are those who feel uncomfortable about the use of assumptions in models because they deem assumptions (in any form) to be too simplistic and improbable. But whether they realize it or not, we all use assumptions to solve problems even those related to our normal, everyday lives. If assumptions were not used, we would be perplexed into indecision by the complexity of a system because there are just too many factors that can affect the outcome of the system.

Using key assumptions simplify our understanding of the system and allow us to build a model to solve problems. As we will later see, it is vital that we test our mathematical models against reality to ensure that our models are accurate despite the use of assumptions.

3) Model simplicity versus model accuracy

In the development of a mathematical model, there is often a trade-off between model simplicity and model accuracy (Fig. 1.1). The more holistic and in-depth we want to represent the system, the more complex and accurate our model becomes. It becomes increasingly more accurate because increasingly more information about the system is summarized and fewer assumptions are used in the model. In contrast, the simpler the model, the less accurate our model becomes because the model tells us increasingly less about the system, resulting in greater loss of information.

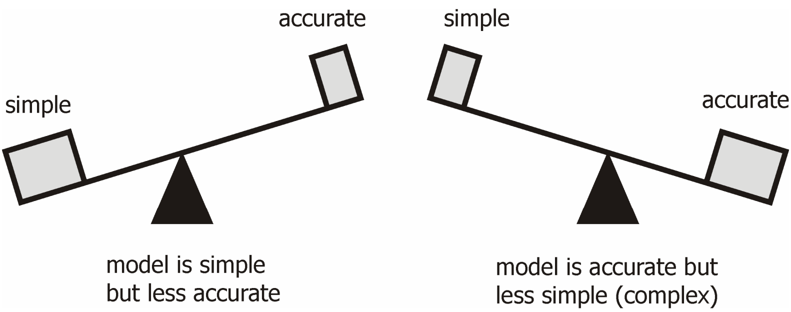

Fig. 1.1. There is often a trade-off between model simplicity and accuracy

Complex models, however, are often cumbersome to use, often requiring intensive and repetitive calculations and a lot of data, some of which may not be easily available. In contrast, simpler mathematical models are less cumbersome to use, requiring not only less but more easily available data.

How we decide between a complex and simple model depends very much on the interests and purpose of our study. At times, only a rough estimation will suffice, so we will be better off with a simpler model. But when high accuracy is required or in critical operations (*e.g.*, the development of a new type of airplane), the use of a complex, accurate model is mandatory.

4) No one best model for all circumstances

A real system is often described by many alternative models. This is because one model may focus on one aspect of the system, not covered or covered with insufficient depth by the other models. These models also differ from each other in terms of their model simplicity and accuracy (Fig. 1.1). Consequently, there is no one model that is suitable for all circumstances. The best model is one that meets our interests and purpose of study, not necessarily always being the most accurate or most simple model. Depending on our objectives, one model (say, model A) could be chosen over the others because the chosen model best meets our